\textbf{À TROUS WAVELET DECOMPOSITION APPLIED TO DETECTING IMAGE EDGE}

Xiaodong ZHANG \textsuperscript{1} Deren Li \textsuperscript{1}

(National Laboratory for Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, 129 Luoyu Road, Wuhan, China
E-mail: xdzhang@hp01.wtu.cn)

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\textbf{ABSTRACT}

An image edge can be defined as a difference of image features in a local image region, its appearance is like as a mutation of different image gray-level or texture structures or image colors. Image edges are very important to human being and in computer machine vision field, because the image edges can transfer the most information of an image. Detecting image edge is considered as a key step in many complicated processing methods such as image segmentation, image recognition, and feature extraction. Detecting edge is an unsolved problem in the image process field. Hitherto, many methods of detecting image edges have been developed, but almost every method has its restriction in application of image processing. In this paper, disadvantages and advantages of some classic methods for image edge detection are thoroughly discussed. Base on the analysis of the classic methods, a new à trous wavelet decomposition algorithm is applied to detecting image edge in the paper, characteristics of the algorithm are discussed in detail. The à trous decomposition is one of discrete wavelet transform algorithms, from the à trous wavelet decomposition theory a detecting image edge method is derived. According to the à trous wavelet decomposition theory, an image can be decomposed into some wavelet planes of increasing scales, and the wavelet planes have the same number of pixels as the original image. That is very important to some applications, for example image fusion and image classification. Therefore, an algorithm of detecting image edge is derived from the à trous wavelet decomposition. The algorithm is suitable for computer program and can execute efficiently. We use the method, classic Sobel, and Robert algorithms to process one SPOT image. From the process results, we can find that the main edges detected by our à trous wavelet decomposition method are better than those processed by classic Sobel, and Robert methods. We also note that the new algorithm of detecting image edge can gain more tiny fine edge information than the other two classic methods. In addition, when the original image is stained by noise, the detecting image edge algorithm developed on the à trous wavelet decomposition almost is not disturbed, on the contrary, the classic Sobel, and Robert algorithms are sensitive to noise.

In order to test robust and flexibility, in practice, we also have employed the algorithm of detecting image edge developed in this paper to produce a binary image. The experiment results are shown the algorithm is satisfactory. Therefore, we think that the algorithm is superior to the other two traditional methods in many aspects, especially when it is applied to those original images which are stained by noise.

However, besides the advantages of the new method of detecting image edge, we also find out some shortcomings, for example the contrast of result image is weaker, so that the detected image edges is not more evident, etc. Those disadvantages need to further research for improving the result.

1. Introduction

Image edge can be defined as the difference of image features in a local region. Its appearance is the mutation of image gray or texture structure or color. The image edge is very important to both human being and machine vision because it can describe the shape of a region, define local feature and convey most information in an image. Detecting image edges is considered as a key step in many complicated processing methods such as image segmentation, image recognition and feature extraction.

There are some classic operators which are used to detect image edges, like gradient, Laplace, LOG, Sobel, Prewitt and Robert operators etc. Gradient operator looks like a high pass filter, but it only sharpens image edges. Some experiments have proved that the methods based on the difference are not effective to detect complicated image edges; The Sobel method is a weighted average operator, it contributes weight to the center pixel in order to enhance the edges; Robert operator is sensitive to noise, therefore it is seldom used in the dense point region; The Laplace
operator is invariant in different directions, it means you can get the same detected edges when you rotate the operator. The Laplace operator not only response to image edges, but also to corner points, end points of a line and isolated points. To restrain noise, the LOG operator smooths an image at first, then performs the difference. Though the image noise is partly restrained, the detected result is influenced. From above discussion, we can find the operators which are relate to direction are not effective to detecting edges when the image is complicated and there are abundant edges in the image. In addition, due to noise in an image the detected result is not perfect if the operators are directly used. Therefore in this paper, our objective is to introduce a new method to detect image edge so as to get a better result.

2. The A Troux Wavelet Decomposition

2.1 Wavelet Decomposition

Wavelet analysis has been successfully used in image processing field. Wavelet transform is a new method which can decompose an image into different resolution images. Suppose function $\psi(x) \in L^2(\mathbb{R})$ is quadratically integrable space, and $\hat{\psi}(\omega)$ satisfies:

$$C_{\psi} = 2\pi \int_{-\infty}^{+\infty} |\hat{\psi}(\omega)|^2 d\omega < +\infty$$

(1)

or

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0$$

(2)

Where $\hat{\psi}(\omega)$ is the Fourier transform result of $\psi(x)$.

When $\psi(x)$ satisfies equation (1) or (2) and can rapidly converge, $\psi(x)$ is called basic wavelet. $\psi(x)$ flexes a and shifts b then we have:

$$\psi_{a,b}(x) = |a|^{1/2} \psi \left( \frac{x-b}{a} \right)$$

(3)

Where $a, b \in \mathbb{R}, a > 0$ and $\psi_{a,b}(x)$ is called wavelet.

Wavelet transform of distribution function $f(x)$ can be defined as following:

$$\psi f(a,b) = \int f(x) \psi_{a,b}(x) \, dx$$

(4)

Where $f(x) \in L^2(\mathbb{R}), b \in \mathbb{R}, \psi_{a,b}(x)$ is complex conjugate of $\psi(x)$.

2.2 The A Troux Wavelet Decomposition Method

Continuous wavelet transform definition is introduced above. It can't be used in computation by program. To do it by computer, we must make the continuous wavelet transform discrete. There are many methods to perform discrete wavelet transform, such as pyramid algorithm which make use of orthogonal basis to decompose an image (or a signal), but the dimension of the result image is changed, that is not advantageous in some applications like pattern recognition and image fusion etc.

To get the result image of the same dimension as the original one, we adopt the algorithm: a trous algorithm. The discrete approach of the wavelet transform can be done with the special version of the so-called a trous algorithm (with holes). The algorithm can decompose an image (or a signal) into an approximate signal and a detail signal at a scale, the detail signal is called a wavelet plane, which is the same as the original image in dimension.

Suppose that the sampled image data $C_0(x)$ are the scalar products at pixels k of the function $\phi(x)$ with a scaling function $\phi(x)$ which corresponds to a low pass filter. The first filtering is then performed by a twice magnified scale leading to the $C_0(x)$ set. The image difference $C_0(x)-C_0(x)$ which is called first wavelet plane contains the information between these two scales and is the discrete set associated with the wavelet transform corresponding to $\phi(x)$. The associated wavelet is therefore $\psi(x)$.

$$\frac{1}{2} \psi \left( \frac{x}{2} \right) = \phi(x) - \frac{1}{2} \phi \left( \frac{x}{2} \right)$$

(5)

The distance between samples increasing by a factor 2 from the scale $(i-1)$ ($i>0$) to the next one, $C_i(x)$ is given by:

$$C_i(x) = \sum_{l} h(l) \cdot C_{i-1}(x + 2^{-i} l)$$

(6)

and the discrete wavelet transform $w(x)$ by:

$$w_i(x) = C_{i-1}(x) - C_i(x)$$

(7)

Where $w(x)$ is the wavelet coefficient and $C(x)$ is approximate signal at the i scale, $h(l)$ is a low pass filter.

The coefficients $h(l)$ is derived from the scaling function $\phi(x)$:

$$\frac{1}{2} \phi \left( \frac{x}{2} \right) = \sum_{l} h(l) \cdot \phi(x - l)$$

(8)

The algorithm can be used to rebuild the data frame because the last smoothed array $C_n$ is added to all the differences $w_i$.

$$C_0(x) = C_{np}(x) + \sum_{i=1}^{np} w_i(x)$$

(9)

If the linear interpolation for the scaling function $\phi(x)$ (see figure...
1) is chosen
\[ \phi(x) = \begin{cases} 1 - |x| & \text{if } x \in [-1,1] \\ 0 & \text{if } x \notin [-1,1] \end{cases} \]

At each scale \( j \), we obtain a set of \( w(x) \) (we also call it wavelet plan) which has the same number of pixels as the image.

If a B-spline for the scaling function is chosen, the coefficients of convolution mask in one dimension are \((1/16, 1/16, 1/4, 1/4, 1/16)\), and in two dimensions

\[
\begin{pmatrix}
1 & 1 & 1 & 3 & 1 & 1 \\
256 & 64 & 128 & 64 & 1 & 1 \\
1 & 1 & 3 & 1 & 1 & 1 \\
64 & 16 & 32 & 16 & 64 & 1 \\
3 & 3 & 9 & 3 & 3 & 3 \\
128 & 32 & 64 & 32 & 128 & 1 \\
4 & 4 & 8 & 4 & 4 & 4 \\
256 & 128 & 64 & 64 & 256 & 1
\end{pmatrix}
\]

3. Detecting Edges Using the A Trous Algorithm

From above analysis, it is found that an image can be decomposed into several wavelet planes at different scales, some of high frequency information is included in the wavelet planes. Therefore we can detect edges of a remote sensing image by using the following procedure:

1) Initialize \( i = 0 \), and input an original image \( f(x,y) \).
2) Convolute the image with the low pass filter \( h(x,y) \)
   \[ f_1(x,y) = f(x,y) * h(x,y) \]
3) Get a wavelet plan
   \[ w_1(x,y) = f_1(x,y) - f_0(x,y) \]
4) If \( i < n \) (the \( n \) is defined the decomposition number)
   then \( i = i+1 \), and return to the step 2; otherwise, stop.
5) Repeat the steps 2, 3, 4) until \( i = n \).

In order to process the image borders, the mirror symmetry method is adopted, namely,

In the row direction:
\[ f(i, j) = f(i, j) \]
\[ f(i+k, j) = f(i, j) \]

Where \( i \leq N, k = 1, 2, ..., N \) is the total rows of the image.

In the column direction:
\[ f(i, j) = f(i, j) \]
\[ f(i, j+k) = f(i, j) \]

Where \( j \leq N, k = 1, 2, ..., N \) is the total columns of the image.

4. Discussing and Conclusions
The figure 3(a) is a SPOT panchromatic image acquired from Wuhan. There are obvious objects such as roads and some fields in the image. The figure 4(a) results from figure 3(a) stained by random noise. The figure 3(b), 3(c) and 3(d) are the results which are respectively processed by the classic Sobel, Robert operators and the à trous wavelet decomposition method. In the condition of noise, we use the methods and get the results shown in figure 4(b), 4(c) and 4(d). Comparing with the figure 3, we can find the main edges of the figure 3(d), for example, the two roads are finer than those in the figure 3(b) and 3(c). The tiny field edges are obvious in the figure 3(d), but they do not exist in the figure 3(b) and figure 3(c). From the results of figure 3, the edges detected by the proposed method are finer and it can detect the tiny edges.

From the figure 4(b) and 4(c), we find the random noise effect is apparent, but almost no the noise effect in the figure 4(d). Therefore the classic Sobel and Robert methods can not filter noise, they are sensitive to noise. The à trous wavelet decomposition method can detect edges and filter random noise. The major reason may be that the big features of the image almost don't verify at the different scales of wavelet transform, but the random noise rapidly attenuates with the increasing scales; on the other hand, the random noise is weakened by adding some wavelet planes. Therefore, selecting proper scales, we can detect the edges and overcome random noise. In some applications we have used the results detected by the à trous wavelet decomposition method and the results are satisfactory.
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References: