8 values of bZ which are plotted as ordinates on graph paper at an enlarged scale.

If the photographs are correct the by and bz coordinates locate points on the same straight line.

An abnormal y-parallax in one of the points A or A' is indicated by a failure of alignment of the corresponding bz, the by having been aligned.

An anomalie in one of the N points is indicated by an opposite error of alignment of the two points at A and A'. By introducing by in the system as shown by the lack of alignment on the graph, and removing the parallaxes in these points by bz, one can prove that the same value of by will help both points, if the light rays of these points are not themselves affected by the deviation.

The y-deviation of one or several perspective rays of points of orientation causes a failure in the alignment of the by and bz coordinates of the graph and β-error will remain. This error introduced in the by of other planes a failure of alignment function at different distances z_0 from the points N, and in the bz a failure of alignment function in distances z and y; they make impossible the total removal of parallax from all the points of a plane with only the bz motion. The plane of orientation should be abandoned and another one used.

In resumé, it is possible, by the preceding simple and rapid method to diagnose the transverse deviations of perspective rays detrimental to relative orientation. The points affected by these deviations must not be used in the relative orientation. The parallaxes that remain in the zones of deviation must be removed locally by by when one wants to see the image of these zones stereoscopically.

The corrections of parallax by by and bz must be repeated several times to reduce the accidental errors; the values used are the average of the results obtained at each point.

IDENTIFICATION AND CORRECTION OF THE EFFECTS OF LATERAL LOCAL DISTORTIONS IN THE PERSPECTIVE BUNDLES IN STEREO TRIANGULATION

by

George Poivilliers.

Abstract. The lateral local distortions of the perspective bundles cause linear errors of points in pairs of photographs in stereo triangulation. These distortions can be analyzed and their effect can be eliminated.

In a recent article I indicated the existance of abnormal local distortions of perspective rays due particularly to asmospheric refraction. I showed that the effects of the y-parallax of these deviations were related to the relative orientation of the stereoscopic model and how, by an appropriate method, it was possible to diagnose these transverse deformations and to eliminate their effect.

The y-component changes the stereoscopic parallax and also the heights z of the terrain mapped. In an isolated pair, the error dz which results can be detected only on known control points, and then only where the number of those points is superabundant.

We now proceed to show that in photogrammetric stereotriangulation it is possible to detect those points in the overlap area where such distortions exist.

In review α', β', γ' and α'', β'', γ'' are the corrections to the angular settings of a pair projectors about the *x*, *y* and *z* axes of orientation, δ_z' and δ_z'' the errors of the *bz* settings, Δb the error in *bx*, the *by* setting is zero; the elevation errors d_z' and d_z'' of points located respectively in the planes $x=0$ and $x=b$ can be expressed as:

$$d_z' = \frac{z}{b}\Delta b + \delta_z'' + \frac{b^2+z^2}{b}\alpha'' - \frac{z^2}{b}\alpha' + y\beta'' - \frac{yz}{b}(\gamma''-\gamma'),$$

$$d_z'' = \frac{z}{b}\Delta b + \delta_z' + \frac{z^2}{b}\alpha'' - \frac{b^2+z^2}{z}\alpha' + y\beta' - \frac{yz}{b}(\gamma''-\gamma').$$

In a model $(k-1)$ the points that are common with the model k lie in the plane $x=b$ and in the model k these same points lie in the plane $x=o$. The height difference Δz_k that a point has in passing from one model to the next can be expressed as

$$\Delta z_k = z_k - z_{k-1} = \frac{z}{b}(\Delta b_k - \Delta b_{k-1}) + \delta_z'' - \delta_z' + b(\alpha''+\alpha') + y(\beta''-\beta^1)$$

if the base b is the same in the two models, if the errors of the settings are systematic, *and if none of the corresponding perspective rays are distorted in the y-direction.*

The quantity

$$e = \delta_z'' - \delta_z' + b(\alpha''-\alpha')$$

can be determined in obtaining the summation of $b\Delta z/z$ for all the points N on the plane $y=0$ using common points in the overlap area of two models.

$$\sum_2^n \frac{b}{z}\Delta z = (\Delta b_n - \Delta b_1) + e\sum_2^n \frac{b}{z}$$

or the same summation for all the points A and A' where the ratios of the coordinates y/z are opposite

$$\sum_2^n\left(\frac{b}{z}\Delta z_A + \frac{b}{z}\Delta z_{A'}\right) = 2(\Delta b_n - \Delta b_1) + e\sum_2^n\left(\frac{b}{z}_A + \frac{b}{z}_{A'}\right).$$

The systematic error of the setting $\beta = \beta'' - \beta'$ can be immediately determined from the relation

$$\sum_2^n\left(\frac{b}{z}\Delta z_A - \frac{b}{z}\Delta z_{A'}\right) = e\sum_2^n\left(\frac{b}{z}_A - \frac{b}{z}_{A'}\right) + (n-1)\frac{y}{z}b\beta.$$

The accidental errors of setting and the y-components of the deviations of the perspective rays which modify the warp Δz a little, the theoretical values of e and of $b\,\beta$ due only to systematic errors of relative orientation if, in doing this, one has taken the precaution to eliminate the y-deformations.

In the overlap of two adjacent models, the Δz must then satisfy the relation

$$\frac{b}{z}\Delta z = (\Delta b_k - \Delta b_{k-1}) + \frac{b}{z}e + \frac{y}{z}b\beta$$

and for the points N $(y = 0)$, A $(y/z) = +t$, A' $(y/z) = -t$ the relations

$$\left(\frac{b}{z}\Delta z_A - \frac{b}{z}\Delta z_N\right) = e\left(\frac{b}{z}_A - \frac{b}{z}_N\right) + t\,b\,\beta$$

$$\left(\frac{b}{z}\Delta z_N - \frac{b}{z}\Delta z_{A'}\right) = e\left(\frac{b}{z}_N - \frac{b}{z}_{A'}\right) + t\,b\,\beta$$

must be satisfied with the limits of precision, in one case in the relative orientation procedure and in another case in the measurements of elevations. An error in one of the three is thus easily detected, indicating a y-distortion anomalie of the perspective rays of the corresponding point.

One can perform the solutions graphically by plotting as abscissas the quantities

$$(b/z_A - b/z_{.N}) \qquad \text{and} \qquad \left(\frac{b}{z}_N - \frac{b}{z}_{A'}\right)$$

and as ordinates the values of the function $b\,\Delta z/z$ the points must lie on a line of slope e, whose y-intercept is $t\,b\beta$.

In the case of fairly level terrain the systematic error e is not erratic and one must have

$$\Delta z_A - \Delta z_N = \Delta z_N - \Delta z_{A'} = y\,\beta;$$

the operation of using ground control is simplified.

It is thus possible to eliminate an important cause of observed breaks in stereo triangulation.

METHOD OF PHOTOGRAMMETRIC STEREO TRIANGULATION

by

George Poivilliers.

Abstract. The systematic errors are made as constant as possible by keeping the air base at a fixed length. The errors caused from local distortion anomalies of the perspective rays are eliminated. The operations are reduced to a single adjustment.

Photogrammetric stereo triangulation consists of attaching successive models where the scale and orientation are adjusted to fit the preceding model. The first and last models of the chain are adjusted to points of known ground position. In the "cantilever extension" only the first model rests on known control points.

In this chain there is an accumulation of errors, a failure of the orientation and scale of one model is reflected in all the models that follow.

These failures in the orientation and in the scale of the models and their deformations can have an accidental character, such are those indicated by the