In the overlap of two adjacent models, the Δz must then satisfy the relation

$$\frac{b}{z} \Delta z = (\Delta b_k - \Delta b_{k-1}) + \frac{b}{z} e + \frac{y}{z} b\beta$$

and for the points N ($y = 0$), A (y/z) $= +t$, A' (y/z) $= -t$ the relations

$$\left(\frac{b}{z} \Delta z_A - \frac{b}{z} \Delta z_N\right) = e\left(\frac{b}{z}_A - \frac{b}{z}_N\right) + t b \beta$$

$$\left(\frac{b}{z} \Delta z_N - \frac{b}{z} \Delta z_{A'}\right) = e\left(\frac{b}{z}_N - \frac{b}{z}_{A'}\right) + t b \beta$$

must be satisfied with the limits of precision, in one case in the relative orientation procedure and in another case in the measurements of elevations. An error in one of the three is thus easily detected, indicating a y-distortion anomalie of the perspective rays of the corresponding point.

One can perform the solutions graphically by plotting as abscissas the quantities

$$(b/z_A - b/z_{\cdot N}) \qquad \text{and} \qquad \left(\frac{b}{z}_N - \frac{b}{z}_{A'}\right)$$

and as ordinates the values of the function $b \Delta z/z$ the points must lie on a line of slope e, whose y-intercept is $t b\beta$.

In the case of fairly level terrain the systematic error e is not erratic and one must have

$$\Delta z_A - \Delta z_N = \Delta z_N - \Delta z_{A'} = y \beta;$$

the operation of using ground control is simplified.

It is thus possible to eliminate an important cause of observed breaks in stereo triangulation.

METHOD OF PHOTOGRAMMETRIC STEREO TRIANGULATION

by

George Poivilliers.

Abstract. The systematic errors are made as constant as possible by keeping the air base at a fixed length. The errors caused from local distortion anomalies of the perspective rays are eliminated. The operations are reduced to a single adjustment.

Photogrammetric stereo triangulation consists of attaching successive models where the scale and orientation are adjusted to fit the preceding model. The first and last models of the chain are adjusted to points of known ground position. In the "cantilever extension" only the first model rests on known control points.

In this chain there is an accumulation of errors, a failure of the orientation and scale of one model is reflected in all the models that follow.

These failures in the orientation and in the scale of the models and their deformations can have an accidental character, such are those indicated by the lack of precision in clearing the y-parallax. The accumulation of the errors that they induce act according to unforeseen laws.

The failures caused by the imperfections of the adjustment of the instruments or constant deformations of the perspective projections are of a systematic character. This is more rigorous if the same conditions are applied to relative orientation. It is then possible to establish rules of accumulation that permit the correction of errors that are introduced.

Eventually other failures appear from deformation anomalies of certain perspective rays of the photographs being used; they introduce breaks in the rules of accumulation of systematic errors.

In recent articles we have shown how one can perform relative orientation without guessing and with very good accuracy and how one can diagnose and eliminate the erratic effects of the deformation anomalies of the perspective rays.

The present article is relative to a method of stereotriangulation furnishing rigorous rules of the accumulation of systematic errors.

Operating method, measurements. — The airplane is supposed to follow as straight a course as possible, that is, without sudden changes in heading (crab) or in altitude; the exposure interval is supposed to be constant.

In the transition from model to model in the plotting instrument the base length along the x-axis is clamped at a constant value b, the by and bz components are set at zero. The pass points ANA' are marked on each photograph to help in visualizing the problem. The relative orientation is performed using exclusively the perspective rays corresponding to these same points or at points very nearby; images that have been found to be in an area of distortion must not be used in connecting one model to another.

The measurements to be made are:

The stereoscopic measurement of the x, y, z coordinates of the passpoints ANA';

The measurements of the angles a and i of the marked points (possible only on the Poivilliers stereotopograph type B) and the x, y coordinates of those points that have the same constant z_0 coordinate.

In passing from one model to another, the angle i of the point N is maintained rigorously, or corrected in case of a y-distortion anomalie, or in case of a sudden change in heading (crab).

Computation of the position elements of a model k. — These elements are seven in number, the relative orientation having fixed 5 of the 12 possible adjustments of the two projectors. In all those that follow we take the systematic errors into account. For the definitions of the terms one is referred to the preceding articles.

1. *Length of base* $b_k = b - b_k$. — The difference in distance

$$\Delta z_k = z_k - z_{k-1}$$

of a passpoint between the (k — 1) model and the k model satisfies the relation

$$\Delta z_k = \frac{z}{b}(\Delta b_k - \Delta b_{k-1}) + [\delta_z'' - \delta_z' + b(\alpha'' + \alpha')] + y(\beta'' - \beta')$$

which, applied at the point N $(y = 0)$ and at the points A and A' $(y/z = +t)$, must furnish two equal values of Δb_k

$$\Delta b_k = \Delta b_1 + \sum_2^k \frac{b}{z} \Delta z_N - e \sum_2^k \frac{b}{z}{}_N = \Delta b_1 + {}^1/_2 \sum_2^k \left(\frac{b}{z} \Delta z_A + \frac{b}{z} \Delta z_{A'} \right) - \frac{e}{2} \sum_2^k \left(\frac{b}{z}{}_A - \frac{b}{z}{}_{A'} \right)$$

in which

$$e = [\delta_z'' - \delta_z' + b(\alpha'' + \alpha')] = \left[(\Delta b_1 - \Delta b_n) + \sum_2^n \frac{b}{z} \Delta z_N \right] : \sum_2^n \frac{b}{z}{}_N =$$

$$= \left[2(\Delta b_1 - \Delta b_n) + \sum_2^n \frac{b}{z} \Delta z_A + \frac{b}{z} \Delta z_{A'} \right] : \sum_2^n \left(\frac{b}{z}{}_A + \frac{b}{z}{}_{A'} \right).$$

2. *Inclination s_k of the air base to the horizon and the vertical component bz_k.* — The difference $\Theta = (a_k - a_{k-1})$ of the angular setting of the perspective bundle k contains a constant term α caused by systematic errors of setting the convergence of the verticals at the ends of the base and a variable term ε_k representing the difference in inclination of the two cases $(k-1)$ and k to the horizon: $\Theta_k = \alpha + \varepsilon_k$, where the inclination of the base

$$s_k = s_1 + \sum_2^k \Theta - (k-1)\,\alpha$$

and the value of the vertical component $bz_k = (b - \Delta b_k)\, s_k$. The model n being supposedly supported by known vertical control points, we have

$$\alpha = \left(s_1 - s_n + \sum_2^n \Theta \right) : (n-1)$$

3. *The y-setting φ_k of the k projector.* The systematic error $\beta = \beta'' - \beta'$ of the relative y-tilt setting of the two projectors changes linearly the y-tilt of the model, the correct setting being

$$\varphi_k = i_k - (k-1)\,\beta;$$

one can find β from the relation

$$(n-1)\frac{y}{z} b\,\beta = \sum_2^n \left(\frac{b}{z} \Delta z_A - \frac{b}{z} \Delta z_{A'} \right) - e \sum_2^n \left(\frac{b}{z}{}_A - \frac{b}{z}{}_{A'} \right)$$

or, after rocking the model n on its vertical control points, which furnishes φ_n.

$$\beta = (i_n - \varphi_n) : (n-1)$$

4. *Bearing g_k of the air base.* γ is the systematic error of relative bending, Δx the difference of the abscissas from the same passpoint A or A' having the

constant distance z_0, L the difference of the ordinates of two points in this plane

$$g_k = g_1 + (k-1)\,\gamma + \sum_{2}^{k} (\Delta x_A - \Delta x_{A'}) : L$$

with

$$\gamma = \Big[g_n - g_1 - \sum_{2}^{n} (\Delta x_A - \Delta x_{A'}) : L \Big] : (n-1).$$

5. 6. 7. *Coordinates of the point N_k in the projection system.* Dx, Dy, Dz being the differences in the coordinates of the points N_{k-1} and N_k in the model (k—1) corrected for the systematic errors, the local deformations of the model and for the scale error $\Delta b_k/b$ one has

$$x_k = x_2 + \sum_{2}^{k-1} [(Dx - sDz)\sin g - (Dy + \varphi Dz)\cos g],$$

$$y_k = y_2 + \sum_{2}^{k-1} [(Dx - sDz)\cos g + (Dy + \varphi Dz)\sin g],$$

$$z_k = z_2 + \sum_{2}^{k-1} (sDx + \varphi Dz).$$

Remark. — The method requires only one single adjustment which reduces in half the total time required to perform the operations.

A check for mistakes is furnished by reading all the linear and angular measurements twice.

The final adjustment can be checked on known ground control points in the last model and in the common points in the side lap area of two adjacent strips of photography.

The method applies to extensions by assuming that the hypotheses

$$\Delta b_n = b_1, \qquad \Delta s_n = s_1, \qquad g_n = g_1, \qquad \ldots$$

are true.

It is not imposed that the last model should connect to the reference system of the first model.