General Direction of Cadastral Survey and of the Technical Services of Finance Department

# On the final resting of contours in air surveyed maps on large scale. 

## Communication of Prof. Eng. ALFREDO PAROLI

to the 7th International Congress of Photogrammetry at Washington, 1952.

The accuracy of contour lines in air suerveyed maps on large scale (from $1: 5000$ to $1: 500$ ) can be verified by directly measuring intersections of the ground (using the tacheometer and the stadia) and comparing the profiles thus obtained with the corresponding ones derived from the map. The differences of height of corresponding spots in the two profiles must not exceed the limits of error which are preliminarly established and vary according to the slope of the ground and to the map scale.

From a technical point of view, no objection can be made practically on regard of the above-said testing system; notwithstanding it is resulted too slow and too laborious and permits to ascertain the accuracy of contours of a small zone only, i. e. in proximity of the points, where the above mentioned intersections cut the contour lines.

The search after other testing systems allowing a quicklier, less laborious and more extended verification of mapped contours appears therefore obvious as well as opportune.

A testing system we have experimented since a long time and which resulted simple, quick and very efficient, is the «method of the second plotting». It requires office work only. The method means - as the denomination says that during the plotting of contours or during the operations for final testing of the map, tracing of an adequate number of contour lines must be repeated.

The second plotting carried out by means of the same or by an analogous apparatus like that used for the first plotting, permits, first of all, an immediate demonstrative verification of the degree of accuracy and can further be used for establishing the acceptability of the map.

In fact, if the case may be that both the tracings would be without errors, the two runs obtained for a same contour line by the first and the second plotting will be affected by inevitable errors and will show, therefore, reciprocal planimetric divergencies. The magnitude of these divergencies permits to deduce the total mean error of the operation consisting of the two plottings considered together.

If the second plotting (for verification) is carried out with such care that it may be considered free from errors or at least affected by very small and therefore neglectable errors, the magnitude of the above mentioned divergencies will only depend upon the error due to the original plotting. If, therefore, such divergencies remain between the fixed limits of error, the map may be considered acceptable; if the case is on the contrary, the map cannot be accepted and must be refused or corrected.

The following proceeding can be applied for determining the maximum amount of the allowable planimetric divergency between the two runs of a contour line obtained by the first original plotting and the second control plotting.

We will consider an infinitesimal tract of a generic contour line $C_{1}$ calling $L_{i}$ its mean planimetric distance (at the fore-said tract) from the two adiacent contour lines (the preceding contour line $C_{2}$ and the following one $C_{3}$ ) and $\alpha$ the angle of slope of the ground in the considered tract.

In this case it is possible to demonstrate (1) that the divergency $l_{i}$ between the first run $C_{1}^{\prime}$ and the second run $C^{\prime \prime \prime}$ of the contour line $C_{1}$ must correspond in the considered tract to the condition

$$
\begin{equation*}
\frac{l_{i}}{L_{i}} \leq \frac{2,64}{E} M \tag{1}
\end{equation*}
$$

in which $M$ is the mean plotting error of the contour line at given slope of the ground and at the considered map scale; $E$ is the contour interval.

Further, the contour line $C^{\prime \prime}$, plotted for verification, obviously must not touch the adiacent contour lines $C_{2}, C_{3}$ and also not overstep - on the one or on the other side - the strip formed by the latter two lines; therefore

$$
\begin{equation*}
\frac{l_{i}}{L_{i}} \leq \frac{2,64}{E} \quad M \leq 1 \tag{1'}
\end{equation*}
$$

For the usual flying heights above the ground, the following values, obtained in a theoretic and experimental way (2), can ben attributed to $M$ :

For scale 1:2000 (usual flying height 2000 m )

$$
\begin{equation*}
M_{1}= \pm \sqrt{0,50+1,20 \operatorname{tg}^{2} \alpha} \tag{2}
\end{equation*}
$$

For scale 1:4000
(usual flying height 3000 m )

$$
\begin{equation*}
M_{2}= \pm \sqrt{0,90+3,14 \operatorname{tg}^{2} \alpha} \tag{2'}
\end{equation*}
$$

And furthermore we have, obviously: $\quad \operatorname{tg} \alpha=\frac{E}{L_{i}}$
Let us now suppose, on the other side, that the contour interval $E$ would have been chosen not only with regard to the scale and to the use to be made of the map, but also according to the medium slope of the ground in the considered zone and let us apply the terms indicated in an other communication to
$\qquad$
(1) see A. Paroli - The mean error in plotting contours for air photogrammetric mapping - in «Rivista del Catasto e dei Servizi Tecnici Erariali» Rome, 1940.
(2) see A. Paroli - The altimetric mean error in the aerophotogrammetric maps of the new Italian Cadastral Survey - Communication to the 5th International Congress of Photogrammetry - Rome, 1938.
this Congress, i. e. considering the mean error of tracing. According to these terms the maximum contour interval is given by

$$
\begin{equation*}
E_{0}=2,64 \mathrm{M} \tag{4}
\end{equation*}
$$

As in this case the second inequality $\left[1^{\prime}\right]$ is always fulfilled, from the first one can be obtained, introducing values [2] or [2'], [3] and [4]:

$$
\begin{aligned}
& l_{i} \leq 2,64 \quad \sqrt{1,20+0,50 \frac{L_{i}{ }^{2}}{E^{2}}} \quad \text { (for scale 1:2000) [5] } \\
& l_{i} \leq 2,64 \sqrt{3,14+0,90 \frac{L_{i}{ }^{2}}{E^{2}}} \quad \text { (for scale 1:4000) [5'] }
\end{aligned}
$$

Applying the sign of equality, the above expressions furnish the maximum values allowable for the divergencies $l_{i}$ (i. e. the relative limits of error), referred to the medium distance $L_{i}$ between the considered contour line and the two adiacent ones, in the surroundings of each single element of the contour line itself.
The above mentioned limits of error have been calculated for exemplification and were collected in the following tables regarding contour intervals cort responding to multiples of a thousandth of scale number.

TABLE I
Scale 1: 2000 (flying height above the ground 2000 m )

| Contour Interval$\mathrm{E}=2 \mathrm{~m}$ |  | Contour Interval$\mathrm{E}=4 \mathrm{~m}$ |  | $\begin{aligned} & \text { Contour Interval } \\ & E=8 \mathrm{~m} \end{aligned}$ |  | Contour Interval$\mathbf{E}=16 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathbf{1}_{i} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathbf{l}_{\mathbf{i}} \\ \mathrm{mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathbf{l}_{\mathbf{i}} \\ \mathrm{mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathbf{i}} \\ \mathrm{mm} \end{gathered}$ | $\begin{gathered} \mathrm{l}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ |
| 20 | 18,5 | 6 | 3 | 6 | 2 | 6 | 1,5 |
| 18 | 17 | 5 | 2,5 | 5 | 2 | 5 | 1,5 |
| 16 | 15 | 4 | 2,5 | 4 | 1,5 | 4 | 1,5 |
| 14 | 13 | 3 | 2 | 3 | 1,5 | 3 | 1,5 |
| 12 | 11 |  |  |  |  |  |  |
| 10 | 9,5 |  |  |  |  |  |  |
| 8 | 7,5 |  |  |  |  |  |  |
| 6 | 5,5 |  |  |  |  |  |  |
| 5 | 5 |  |  |  |  |  |  |
| 4 | 4 |  |  |  |  |  |  |
| 3 | 3 |  |  |  |  |  |  |

TABLE II
Scale 1: 4000 (flying height above the ground 3000 m )

| Contour Interval$\mathrm{E}=4 \mathrm{~m}$ |  | Contour Interval$\mathrm{E}=8 \mathrm{~m}$ |  | Contour Interval$\mathrm{E}=16 \mathrm{~m}$ |  | $\begin{gathered} \text { Contour Interval } \\ \mathrm{E}=32 \mathrm{~m} \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{l}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} 1_{i} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{l}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{l}_{\mathrm{i}} \\ \mathrm{~mm} \end{gathered}$ |
| 20 | 12,5 | 6 | 2 | 6 | 1,5 | 6 | 1 |
| 18 | 11,5 | 5 | 2 | 5 | 1,5 | 5 | 1 |
| 16 | 10 | 4 | 1,5 | 4 | 1,5 | 4 | 1 |
| 14 | 9 | 3 | 1,5 | 3 | 1,5 | 3 | 1 |
| 12 | 7,5 |  |  |  |  |  |  |
| 10 | 6,5 |  |  |  |  |  |  |
| 8 | 5 |  |  |  |  |  |  |
| 6 | 4 |  |  |  |  |  |  |
| 5 | 3,5 |  |  |  |  |  |  |
| 4 | 3 |  |  |  |  |  |  |
| 3 | 2 |  |  |  |  |  |  |

As results evident, the use of tables is the simplest one and it is neither necessary to know the slope of the ground nor to proceed to any calculation. It is sufficient to measure graphically - at the various corresponding points the divergency between the second plotting of the contour line and its first tracing; further the medium distance $L_{i}$ between the same contour line and the two adiacent ones is to be measured. The knowledge of this distance is necessary, in order to can read off the allowable error $l_{i}$ from the table established for the relative scale and flying height.

The above described proceeding has been applied in Italy - together with that consisting of the survey of profiles of the ground - for the verification of contours contained in the air surveyed maps of the Italian Cadastral Survey. Besides the advantages of easy application and celerity, it permits to verify large groups of contour lines with relatively low expenses as no measurements on the ground are needed.

The limits of error indicated in the two tables are referred to verifications to be carried out during the plotting operation, as in this case the orientation of photographs is varied between the first plotting and the following control plotting.

Therefore, if the second plotting is carried out after some time for the final testing of the map, setting of the pairs of photographs in the plotting apparatus must be repeated and it will be necessary, in such a case, to take into acount also the (doubtless very little) differencies between their new orientation and the former one they had during the first plotting. This can be done adequately increasing the maximum errors $l_{i}$ indicated in the two tables

It would be possible, of course, to apply an analogous proceeding also for testing of the accuracy of planimetry. However, in this case the verification would be less precise as the fidelity of planimetry not only depends on the degree of accuracy of the plotting, but is also and mainly connected with the accuracy
reached in identifying on the ground the features to be represented in the map on large scale (boundaries of landed property, borders, demarcations, etc.).

Due to the higher degree of accuracy requested for cadastral mapping of planimetry, we should get, of course, embarassed in ascertaining eventual divergencies between the first and the second plotting ol planimetry, as in such cases an apposite reconnaissance in the field would be necessary, in order to establish which of the two drawings obtained for a same planimetric line is the right and which the faulty.

In this way, the utility of the described method should become considerably diminished with regard to the verification of planimetry. On the other hand it results particularly fit for the testing of contours.

Rome, July 18th, 1952.
Prof. Eng. A. PAROLI
Chief of Air Surveying Service

# Sur la vérification de l'altimétrie aérophologrammétrique dans la carlographie á grande ézhelle. 

Résumé de la communication par M. le Prof. Eng. Alfredo Paroli.

La vérification de l'altimétrie représentée sur le plan cadastral est généralement effectuée au moyen de la confrontation direct entre le plan et le terrain, c'est à dire en relevant le profil de ce dernier par de spéciales intersections, et en le comparant avec le profil correspondant repris graphiquement de la carte.

Toutefois, ce procédé consent normalement de contrôler un nombre limité de points, et s'il est étendu à una vaste superficie, il exige une perte sensible de temps, et de frais.

Il est donc utile de se valoir, dans l'exécution de la vérification, de contrôles plus rapides et plus économiques, et spécialement de la méthode de la seconde restitution.

Cette méthode consiste à tracer une seconde fois des portions de l'altimétrie photogrammétrique au moyen des mêmes photogrammes, moyennant lesquels a été effectuée la restitution originale. Les courbes de niveau ainsi tracées à titre de contrôle présentent des écarts plus ou moins sensibles par rapport à la marche obtenue pour elles dans la primitive restitution du plan.

L'entité des écarts sus-indiqués consent de s'assurer si la représentation altimétrique reste, ou non, dans les limites d'erreur fixées. On peut même établir analytiquement d'une façon rigoureuse les écarts maxima admissibles entre le tracé original et celui de contrôle, de manière à rendre applicable pratiquement et avec la plus grande simplicité cette méthode de contrôle.

