# DEM'S COMPARISON FOR THE EVALUATION OF LANDSLIDE VOLUME

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#### **ABSTRACT**

Evaluation of the amount of material displaced has been a problem came out on the occasion of the landslides that struck some countries of region Campania on May 1998.

Landslide volume has been obtained as difference between the Digital Elevation Model derived from the survey carried out on the ground and that one derived from maps of the same zone preceding the event.

Several methods can be applied for the creation of the model and the results are strongly influenced by the choice of the algorithm used. In this work some different interpolators have been tested in order to generate the DEMs of the surfaces with regard to three of the abovesaid landslides. The evaluation of the volume has been carried out by means of the comparison of the two "best" DEMs.

## 1. INTRODUCTION

On the occasion of the landslides occurred in Campania on May'98, the "Protezione Civile" Department organised a series of researches useful for a complete knowledge of the territory.

The topographical research-unity involved in works had also the task to provide some information on the volume of the displaced mud. Such volume can be obtained by means of the difference between the Digital Elevation Model (DEM) achievable from the survey made on the ground after the event and the other one concerning the situation of the same zone before the landslide.

Different techniques and methodologies were adopted for the survey because of the different morphology of the ground: kinematic and differential GPS, total stations both classic and special, such as the one called MDL, that supplies the distance even without the use of reflecting prism.

The ground surveys, to a scale of 1:1000-1:2000, have been performed as from some vertices belonging to a GPS frame network which also include four vertices of the IGM95 national network. By means of such vertices, known in the different geodetical system, the co-ordinates of the surveyed points has been transformed in the National cartographic system Gauss-Boaga.

In the same cartographic system have been acquired the co-ordinates of the contour lines with equidistance equal to 25 m from the elements of the "Cassa del Mezzogiorno" cartography at scale 1:5000, produced in 1974 (we shall subsequently use the acronym CdM).

The acquisition of such co-ordinates passes through a procedure of semiautomatic digitization of the contour lines coming from the cartography in raster form, previously geo-referenced to mean of a similar transformation.

The contour line points yield a Digital Elevation Model that can be compared with the one derived from the survey, provided that both have the same step and size.

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The software of elaboration of the DEMs used for the experimentation is the Surfer, of the American House "Golden Software", version 6.04 of 1996. It is a software that elaborates numerical data (typically co-ordinates of points in the space) furnishing models of surface on square grid. It foresees the most common modality of data interpolation.

The tests have been carried out on three landslides, called SA20, SA30 and SA60.

## 2. USED INTERPOLATION'S ALGORITHMS

The gridding methods included with Surfer can be divided into two general categories: exact interpolators and smoothing interpolators. Some exact interpolators can incorporate a smoothing factor that causes them to become smoothing interpolators.

Exact interpolators honour data points exactly when the data point coincides with the grid node being interpolated. Even when using exact interpolators it is possible that the grid file does not exactly honour the data if points do not coincide with the grid nodes.

In the next the griddings algorithms used for the tests are described; the abbreviate word used sometimes in the next is written between brackets.

# 1. Inverse Distance to a Power (I.D.)

It is a weighted average interpolator, and can be either an exact or a smoothing interpolator.

The power parameter controls how the weighting factors drop off as distance from a grid node increases. The weight given to a particular data point when calculating a grid node is proportional to the inverse of the distance to the specified power of the observation from the grid node. When calculating a grid node, the assigned weights are fractions, and the sum of all the weights is equal to 1. When an observation is coincident with a grid node, the observation is given a weight of essentially 1, and all other observations are given a weight of almost 0. In other words, the grid node is assigned the value of the

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coincident observation. This is an exact interpolator.

The power parameter determines how quickly weights fall off with distance from the grid node. As the power parameter approaches zero, the generated surface approaches a horizontal planar surface through the average of all observations from the data file. As the power parameter increases, the generated surface is a "nearest neighbor" interpolator and the resultant surface becomes polygonal. The polygons represent the nearest observation to the interpolated grid node.

One of the characteristics of I.D. is the generation of "bull's-eyes" surrounding the position of observations within the gridded area. It is possible to assign a smoothing parameter during inverse distance gridding, which causes their reduction.

2. Kriging (KR.)

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It is a geostatistical gridding method that produces visually appealing contour and surface plots from irregularly spaced data. KR. attempts to express trends that are suggested in the data so that, for example, high points might be connected along a ridge, rather than isolated by bull's-eye type contours.

3. Minimum Curvature (M.C.)

The interpolated surface generated by M.C. is analogous to a thin, linearly-elastic plate passing through each of the data values with a minimum amount of bending. It generates the smoothest possible surface while attempting to honour data as closely as possible. M.C. is not an exact interpolator however; this means that the residuals are not always small.

4. Nearest Neighbor (N.N.)

The N.N. gridding method assigns the value of the nearest datum point to each grid node. This method is useful when data is already on a grid, but needs to be converted to a Surfer grid file. Or, in cases where the data is nearly on a grid with only a few missing values, this method is effective for filling in the holes in the data.

5. Polynomial Regression (P.R.)

P.R. is used to define large-scale trends and patterns in the data. It is not really an interpolator because it does not attempt to predict unknown Z values.

It is possible to select the different types of polynomials, among the following ones: simple planar surface, bilinear saddle, quadratic or cubic surface.

It is a very fast method for any amount of data, but local details in the data are lost in the generated grid.

6. Radial Basis Functions (R.B.F.)

Radial Basis Functions are a diverse group of data interpolation methods. All of the R.B.F. methods are exact interpolators. It is possible to introduce a smoothing factor to all the methods in an attempt to produce a smoother surface and to specify some functions in order to define the optimal set of weights to apply to the data points when interpolating a grid node.

7. Shepard's Method (SH.)

This method uses an inverse distance weighted least squares method. As such it is similar to the I.D. to a power interpolator but the use of local least squares eliminates or reduces the "bull's eye" appearance of the generated contours. SH.'s method can be either an exact

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or a smoothing interpolator.

8. Triangulation with Linear Interpolation (TR.)

It is an exact interpolator. The method works by creating triangles by drawing lines between data points. The original data points are connected in such a way that no triangle edges are intersected by other triangles. The result is a patchwork of triangular faces over the extent of the grid.

Each triangle defines a plane over the grid nodes lying within the triangle, with the tilt and elevation of the triangle determined by the three original data points defining the triangle. All grid nodes within a given triangle are defined by the triangular surface. Because the original data points are used to define the triangles, data set is honoured very closely and the residuals are small.

TR. works best when data points are evenly distributed over the grid area. Data sets that contain sparse areas result in distinct triangular facets on a surface plot or contour map. TR. is very effective at preserving break lines.

## 3. DEMS INTERPOLATION

Using the set of data obtained from the survey on the ground many tests of interpolation have been carried out with the purpose to choose those more reliable for the elaboration of the DEM.

The faithfulness of the interpolated DEMs has been evaluated through two criteria:

- a "statistical" criteria

a criteria based on the "visual analysis"

The values minimum and maximum of the residuals and theirs statistic parameters, i.e. the average and the standard deviation, have been calculated and appraised. Contour line maps have been outlined to verify adherence of the graph to the real morphology of the ground and to locate, within the interpolated zone, possible zones with anomalous characteristic elements.

For the experimentation a computer compatible IBM from the following characteristics has been used: Intel Pentium III 450 MHz processor; 128 MB RAM.

The first series of interpolations has been carried out with all the methods foreseen from the software SURFER. For all methods the options of default have been set, sort exception for the method P.R. for which as default is defined the plain, while a polynomial function of degree 10 has been chosen.

A rectangular area that includes the surveyed zone has been considered and the chosen grid step has always been equal to 2 m.

In the table 1 the residuals and their statistic parameters for the three landslides are summarised, together with the number of sampled points and the number of points with residuals calculated.

Possible difference between the two values points out the number of knots of the grid in correspondence of which some value of height has not been calculated, because of the lack located of an enough number of points furnished in input. The elaboration time for the creation of the DEM is also reported.

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a in	# of points		918	918	918	918	918	918	918	918
ny dear	# of cells		915	915	915	915	914	915	915	901
03	Average	[cm]	0.5	-0.4	-3.5	0.2	-9.0	-1.2	-6.9	-5.5
SA20	St.Dev.	[cm]	37.9	35.4	48.4	21.0	3490.6	8.9	54.4	43.7
01	Minimum	[cm]	-343.2	-226.5	-370.1	-345.1	-7765.3	-67.7	-444.0	-402.7
	Maximum	[cm]	231.8	289.5	263.4	130.2	7173.0	74.5	249.6	301.0
	Elab.Time	Prince II	1'14"	2'21"	0'1"	0'11"	0'1"	6'16"	1'14"	0'1"
	# of points		997	997	997	997	997	997	997	997
	# of cells		993	993	993	993	993	993	993	982
0	Average	[cm]	1.3	-1.2	0.2	0.0	-18.5	-1.9	-9.0	-6.4
SA30	St. Dev.	[cm]	31.5	31.6	36.0	8.8	2336.4	8.4	46.5	38.3
01	Minimum	[cm]	-123.4	-172.4	-188.1	-106.6	-3754.3	-51.9	-193.1	-366.3
	Maximum	[cm]	151.7	125.8	184.7	82.3	17295.8	38.7	237.3	180.2
	Elab.Time		1'46"	3'15"	0'2"	0'7"	0'1"	6'28"	1'48"	0'1"
	# of points		24869	24869	24869	24869	24869	24869	24869	24869
	# of cells		24858	24858	24858	24858	24858	24858	24858	24792
SA60	Average	[cm]	0.8	1.8	1.3	1.2	0.5	-32.4	0.7	1.4
	St. Dev.	[cm]	68.3	61.7	60.8	65.0	2528.0	2270.3	95.5	70.3
	Minimum	[cm]	-909.3	-778.6	-978.8	-1072.8	-4539.3	-98721.2	-1242.2	-1096.7
	Maximum	[cm]	1166.2	1173.8	1151.1	1281.1	15264.6	143185.4	2659.7	1199.0
the r	Elab.Time		1h10'00"	1 <sup>h</sup> 15'00"	0'6"	3'47"	0'1"	2h05'00"	1 <sup>h</sup> 09'00"	0'1"

Table 1 – DEMs from survey data set. Default options.

Observing the statistic parameters brought in the table 1 it is clear that the results depend partly from the data of input and it is therefore not possible to give an absolute judgement of goodness of an algorithm putting aside from them. Besides, for SA60 all the parameters of interest have higher values.

However, the method Polynomial Regression is revealed to be without doubt the worse one, with unacceptable values of the residuals and standard deviations for all the three landslides.

All the other methods give acceptable values of the residuals and statistic parameters; particularly:

the methods Inverse Distance to Power, Kriging, Minimum Curvature and Triangulation give values of standard deviations nearly from 30 to 50 cm for SA20 and SA30 and from 60 and 70 cm for SA60. The values of maximum and least residual are of the order of the unity of meters for SA20 and SA30 and of the tens of meters for SA60.

- the methods Nearest Neighbor and Radial Basis Function give better results then the precedents for SA20 and SA30. For SA60 the N.N. method furnishes parameters of the same order of greatness of the group of algorithms described previously while the second one furnishes elevated values – not acceptable - of the residuals and of standard deviations, due perhaps to the presence in the data of blunders.

- the Shepard's method, finally, produces worse results slightly of those of the first group for all the landslides, also always acceptable.

Another aspect observable in the table is that relative to the long times necessary to some methods to interpolated the data, especially if they are very numerous. Since one of the free options of the used software of interpolation refers to the choice of the amount and distribution of data taken into account to the goals of the calculation of the co-ordinate Z of the knot (all data or only those ones which are in a certain around, defined from the users), some other tests were carried out varying such parameter for choice a more suitable search radius than the default one. For SA60 a search radius of 100 m does not produce appreciable differences in the values of the residuals in comparison to that of default equal to 3130 m, while the times of elaboration are been reduced sensitively.

As far as it concerns the visual aspect instead, in figure 2 are brought the contour lines maps relative to the DEMs produced from the interpolations with the various algorithms. Only the elaboration carried out for SA20 are presented but the considerations on the results are also valid for the others two landslides. The representation scale is 1:20.000 although the density of the data and the precision of the survey are compatible with representations to a scale of 1:1000. With the dots is marked the limit of the landslide.

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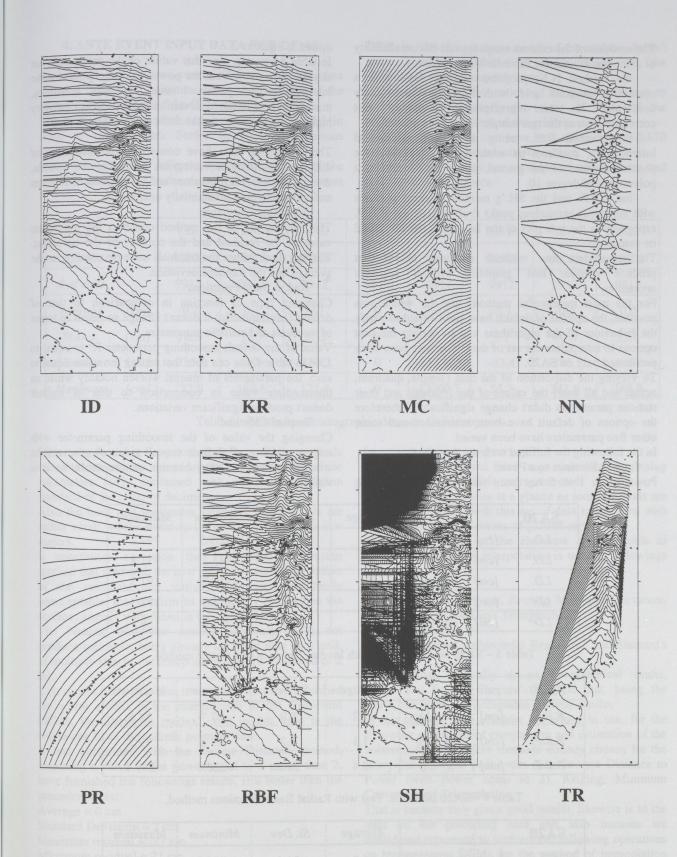


Figure 2 - SA20 landslide (scale 1:20.000). Post event input data.

The analysis of the contour maps reveals the unreliability of some algorithms, for the following reasons:

- the method N.N. produces contour lines with a course to irregular grid, with lines of discontinuity conformed in real precipices, absolutely not correspondents to the real morphology of the ground;

- the method P.R., being a method of global interpolation, confirms its unsuitable to represent the physical surface of the ground, also with the use of a polynomial of degree 10;

the R.B.F. and the SH.'s method produce graphics with numerous anomalous peaks (bull's eyes); the R.B.F. especially in the low part of the landslide, the SH. in all its extension.

The remainders four methods elaborate DEMs that produce contour line graphics without apparent anomalies.

For the most meaningful methods and for which was possible, the options of default have been varied to verify the behaviour of the algorithms and to choose the best options to interpolate the set of data. The tests have been performed only on SA20.

To varying the disposition of the data (simple, quadrant, octant and all data) the values of the residuals and their statistic parameters didn't change significantly; therefore the options of default have been maintained and some other free parameters have been varied.

In the following the fulfilled tests are described.

Inverse Distance to a Power

Power from 1 to 5 has been tested while the default

option is equal to 2.

In table 3 you can note that values of the parameters are slightly dependent from the power, sort exception for the interpolation of power 1 that furnishes minimum, maximum and standard deviation values of residual very high. From 3<sup>rd</sup> power on the differences are very small.

- Kriging

The performed tests have concerned the statement of different functions of variogram. Changing the options, the standard deviation values and the residual maximum and minimum stay substantially unchanged.

- Minimum Curvature

The only option of the method concerns the maximum value of the residual and the number of the iterations. Varying these values the residuals increase notably, as the average and the standard deviation.

Radial Basis Functions

Changing type of function in comparison to that of default (Multiquadric) it doesn't change so much in terms of residuals and statistic parameters.

Value of the default smoothing parameter R<sup>2</sup> is equal to 139; in table 4 you can note that setting the value equal to zero, the parameters of interest worsen notably while an intermediate value in comparison to that of default doesn't produce significant variations.

- Shepard's Method

Changing the value of the smoothing parameter with respect to that of default (equal to zero), the results worsen notably and they become unacceptable, as it is visible in table 5.

SA20			Average	St. Dev.	Minimum	Maximum
I.D.	[cm]	Power 1	5.9	186.4	-1039.5	794.2
I.D.	[cm]	Power 2	0.5	37.9	-343.2	231.8
I.D.	[cm]	Power 3	0.4	23.3	-250.7	191.3
I.D.	[cm]	Power 4	0.4	21.1	-251.5	173.3
I.D.	[cm]	Power 5	0.4	20.4	-253.8	156.1

Table 3 - SA20 landslide. Test with Inverse Distance to a Power method.

SA20			Average	St. Dev.	Minimum	Maximum
R.B.F.	[cm]	$R^2 = 139$	-1.2	8.9	-67.7	74.5
R.B.F.	[cm]	$R^2 = 70$	-1.1	8.2	-65.0	79.2
R.B.F.	[cm]	$R^2 = 0$	-0.4	35.3	-226.6	289.6

Table 4 - SA20 landslide. Test with Radial Basis Functions method.

SA20		1	Average	St. Dev.	Minimum	Maximum
SH.	[cm]	Smooth = 0	-6.9	54.4	-444.0	249.6
SH.	[cm]	Smooth = I	-568.6	1498.3	-11642.8	5354.4
SH.	[cm]	Smooth = 2	-590.8	1602.2	-14394.9	5950.9

Table 5 - SA20 landslide. Test with Shepard's method

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#### 4. ANTE EVENT INPUT DATA FOR DEMS

As said, to get the ante-event DEM we used the data deriving from the digitization of the contour lines of the cartographic elements of the CdM, at scale 1:5000.

The numerical data produced have been furnished in input to the software Surfer to effect the various interpolations.

It is to say that the different spatial disposition of the points in comparison to those surveyed for the post-event

DEM, besides the greatest smooth of the surface, doesn't allow to export to this new source of the data the conclusions of the preceding tests.

The co-ordinates of the vertices of the rectangular area to interpolate are obviously the same of those of the previous tests, as the step of the grid equal to 2 m.

Tests described in the next have been made only on SA20 landslide by using the default parameters.

In table 6 are reported the residuals of interpolation and their statistic parameters.

SA20		I.D.	KR.	M.C.	N.N.	P.R.	R.B.F.	SH.	TR.
# of points # of cells		50508	50508 50508 50508 50508 50		50508	50508 50508		50508	
		9912 99	9912	9912	9912	9912	9912	9912	9912
Average	[cm]	-0.2	0.1	-0.1	0	-3318.4	-0.1	-0.3	0.0
St.Dev.	[cm]	18.3	12.5	21.3	0	7818.2	3.7	40.6	13.6
Minimum	[cm]	-236.4	-170.4	-181.2	0	-15336.5	-53.4	-386.5	-145.3
Maximum	[cm]	172.0	178.6	175.4	0	17200.7	56.7	242.9	169.9

Table 6 - DEMs from cartographic data set. Default options.

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Observing the table 6 we note that almost all the methods introduce maximum values of the residuals and statistic parameters fairly contained with values of deviation standard of the order of decimetres.

Also the least and maximum values of the residuals are rather small, of the order to the maximum one of a few meters.

It constitutes exception the Polynomial Regression method that also with the new set of data introduce values of the residuals and statistic parameters absolutely unacceptable, that confirm its inadequacy to describe the surface, already underlined in the preceding tests.

The Nearest Neighbor method obviously does not produce residuals, being already the input data on a grid, of multiple step of the interpolated one (10 m instead of 2

You can besides note like the method that has furnished the worse results in the interpolation of the post-event DEMs, the Shepard's method, also in this case is the worse one from the statistic point of view.

A test performed with the Inverse Distance method, assuming a value of the power equal to 3 rather than 2, have furnished the followings results, still better then the precedent ones:

Average = 0 cm

Standard Deviation = 2 cm

Minimum residual = -27 cm

Maximum residual = 21 cm

The worse statistic behaviour of the interpolators on the post-event data can be explained partly from the greatest smoothing of the surfaces digitized from cartography and anyway, it is to consider that some out-liers can probably be present in the ground surveys.

In figures 7 the contour lines maps elaborated beginning from the produced DEMs are represented.

First of all, you can note at a glance as some methods are much more similar with this set of data rather then with the ones provided by means of ground survey.

The visual analysis of the elaborate graphic leads to divide the methods of interpolation in the two followings groups:

Acceptable:

Inverse Distance to Power, Kriging, Minimum Curvature, Radial Basis function and Triangulation.

Unacceptable:

Nearest Neighbor, Polynomial Regression and Shepard's method.

This confirms integrally almost the numerical results, except that for the Shepard's Method that, being the worse one, gave also acceptable statistic results.

In this case we also consider opportune to use, for the following operations of comparison and estimation of the volumes, the four select methods already chosen for the interpolation of the post-event data: Inverse Distance to Power (with power equal to 3), Kriging, Minimum Curvature and Triangulation.

That is because they given good results, likewise is to the tests on the preceding data and also because we considered opportune to perform the following operations on homogeneous DEMs for the method of interpolation

The method Radial Basis Functions has not been instead taken in consideration for the respect of the volumes.

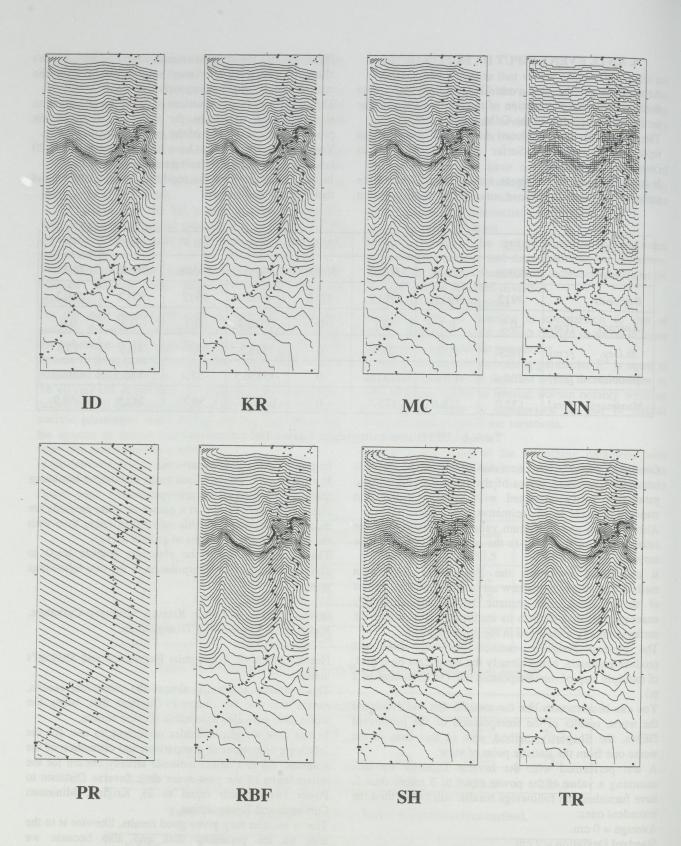


Figure 7 - SA20 landslide (scale 1:20.000). Ante event input data from CdM.

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#### 5. DISPLACED VOLUME ESTIMATION

The volumes are computed through the comparison between the two DEMs, post and ante-event; for volume it is intended that one of prismatic solid inclusive between two surfaces having same area and same grid step. Volumes are calculated using the following three methods:

- Trapezoidal Rule

it is a generalization of the method of the trapezes used for integrating the functions of a variable, in which the area subtended by the curve is equal to the sum of the areas of the trapezes that are obtained from an assigned partition of the interval of integration.

Simpson's Rule

it is also derived by the analogous procedure used for the

functions of a variable in which a function (polynomial of second degree) passing for the gives points is introduced.

- Simpson's 3/8 Rule

it is a variation to the preceding method.

In table 8 a summary of the calculated volumes is carried as to the sum of positive (cuts) and negative (fills) volumes.

The cut volume concerns the material in those places where the upper surface is above the lower surface (zones where the accumulation has been verified) while the fill one is the volume of material in those places where the lower surface is above the upper surface (zones interested from remove of material).

The values of cut and fill volumes carried over are referred to the trapezoidal method and in table they are brought in absolute value

SA20	eddys aks	I.D.	KR.	M.C.	TR.
Trapezoidal Rule	$[m^3]$	-11.292.400	-9.528.890	33.056.200	3.249.320
Simpson's Rule	$[m^3]$	-11.299.200	-9.529.190	33.054.800	3.250.530
Simpson's 3/8 Rule	$[m^3]$	-11.297.000	-9.526.440	33.056.200	3.249.220
Negative Volumes (Fill)	$[m^3]$	12.515.800	10.965.700	10.525.000	1.324.340
Positive Volumes (Cut)	$\lceil m^3 \rceil$	1.223.400	1.436.810	43.581.200	4.573.660

Table 8 - Volumes computation

From the table it is noticed that the differences among the volumes calculated with the three integration methods are very modest. Viceversa, there are great differences among the values relative to the four interpolators, above all as far as it concerns the method M.C. (maximum values) and the TR. (least values), that differ between them of a factor 10.

You note besides like the cut volumes are very different from the fill ones; that is also because in the areas to valley the detritic material has been partially removed in the following days immediately the events. Either for the entity of the values obtained and for the difference between the various methods strong doubts fed on the reliability of the aforesaid values. This induces besides to think that the DEMs (ante and postevent) can not be entirely homogeneous among them because of different references of height.

#### 6. CONCLUSIVE REMARKS

The different spatial disposition of the points causes a different behaviour of the same algorithm.

Summarising the results of the tests turned to a comparison of the most common methods of data interpolation, effected on the base of the evaluation of the residuals as well as the visual analysis of the relative contour line maps, the methods: Inverse Distance to Power, Kriging, Minimum Curvature and Triangulation with Linear Interpolation, can be considered reliable with the set of data at disposal.

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