PROCEDURES OF CORRECTION OF THE GEOMETRY DISTORSIONS FOR DIGITAL IMAGES

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ABSTRACT

Since a few years digital fotocameras have widely spread on the market, principally because of their price, quality image improvement and easy of use. In order to extract from these digital images reliable 3D informations regarding objects position and dimension, both the exact knowledge about perspective and dimensional relations between image and scene, and the knowledge about the geometric distortions parameters are needed.
Therefore, the aim of this work was to develope, in the ambit of the Photogrammetry course, an alternative calibration method and to implement it as teaching software, by which the parameters of the major forms of geometric distortions, i.e. radial, decentering and thin-prism, can be estimated. Basically, the proposed method was realized putting together the Tsai-Lenz and Weng-Cohen-Herniou calibration algorithms and applying the Levenberg-Marquardt algorithm as non linear optimization procedure. The software was developed in Matlab programming language, because its code can be to structured in text-like scripts, allowing therefore to share and to understand a program in easier way compared to software written, fo instance, in Fortran or C.
The method has been implemented in such a way to allow a full camera calibration or a computation of the exterior orientation parameters only, using inner orientation and distortion parameters determined from previous full calibration. This approach can be useful if only an estimate of new targets positions is required, in which the inner and distortion parameters are already given.

1. INTRODUCTION

The camera calibration, typical issue in computer vision, is becoming today very important also in digital photogrammetry applications, where dimensional measurements are required. The main aspect of camera calibration is concerned with the estimate of internal parameters of the camera, that define the perspective and dimensional relations between 3D points in the scene and corresponding image coordinates. Another important aspect of calibration regards the determination of geometrical relations between camera and scene through estimate of external parameters, and the correction of geometrical lens distortions.
On the other hand, digital non-metric fotocameras are widely spreading on the market since a few years, principally because of their price, quality image improvement and easy of use. At the present the image quality of amateur digital cameras is surely not at the same level of film cameras in terms of resolution (300 dpi vs 2500 dpi with a 35mm film), image dimension (640 x 480 pixels), colours spectrum (24 bits, rather than continous) and limited dynamic range of the CCD, in lights and shadows capturing. As rule of thumb, increasing the dimension and the number of CCD cells the image quality is improved, but the final price is augmented as well. Despite the exposed drawbacks, the easy and quickly download of images on a computer and the perspective of improvements in image quality lead to a reasonable scenario in next future, where digital cameras will substitute the film ones in the most common applications.
On the ground of these considerations, we have developed an alternative calibration method of digital non-metric camera, in order to employ this kind of relative expensive device in digital photogrammetry applications.
Following the classification presented in [4], we can identify three main groups for the existing camera calibration techniques:

1) *Direct Nonlinear Minimization*: in this category the parameters estimation involves using an iterative algorithm, which tries to minimize residual errors of some equations. The adopted camera model can be very general, to cover many kind of distortions, but it requires a good initial guess of the parameters because the procedure is iterative. Furthermore including the estimate of lens distortions, the procedure may be unstable, the correlation between external and distortions parameters can lead to divergence or false solutions.

2) *Closed-Form Solution*: parameters values are computed directly through a noniterative algorithm based on a closed-form solution. It is a fast procedure but, in general, camera distortion parameters cannot be incorporated in the algorithm.

3) *Two-Step Method*: this technique involves a direct solution for most of the calibration parameters and some iterative solution for others. An example is the Tsai-Lenz calibration algorithm, where however only the radial distortion is taken into account.

From a short analysis of these calibration methods, one can conclude that noniterative procedures involve closed form solution of linear equations without estimating the distortions parameters, while iterative methods allow to evaluate lens distortion through nonlinear optimization procedures, but they require a good initial guess of the parameters.
In order to solve this trade-off, we addressed the camera calibration problem adopting a two-steps method, based on a combination of the Tsai-Lenz [3] and Cohen-Herniou [4] calibration techniques. In the first step we use the noniterative Tsai-Lenz algorithm to directly compute a closed-form solution for all external and some major internal parameters of a distortion free camera model. In the second step we apply a nonlinear optimization based on a camera model that takes into account various kind of geometrical lens distortions. Because an iterative algorithm is involved, the solution of the first step is used as initial guess.
The main advantages of our method are as follows:

1) Unlike the Weng-Cohen-Herniou method, in the first step we use the well known Tsai-Lenz calibration algorithm, in order to get an initial guess of internal and external parameters. Being noniterative, this algorithm is fast and easy to implement;
2) Compared to Tsai method, in the second step we improve the estimate of *all* camera parameters,
3) Through the application of a nonlinear optimization, we can consider various kind of lens distortion rather than just the radial one;

The method has been implemented in such a way to allow a full camera calibration or a computation of the exterior orientation parameters only, using inner orientation and distortion parameters determined from a previous full calibration. This approach can be useful if only an estimate of new targets positions is required, in which the inner and distortion parameters are already given.
In the following sections a detailed description of the proposed method, so as a short overview of the implemented calibration software, will be presented.

2. THE CAMERA MODEL

In every camera calibration procedure, a certain set of reference systems are required to define the coordinates of target points and of corresponding projections onto the image. In our case we adopted the following set (see Fig. 1):

- $\Sigma_w(O_w, X_w, Y_w, Z_w)$ target fixed 3D reference system, with origin at point O_w; in case of coplanar target points the X_w and Y_w axes are choosen in such a way to set $Z_w=0$.
- $\Sigma_c(O_c, X_c, Y_c, Z_c)$ is the 3D camera fixed reference system; its origin coincides with the optical center of the camera and the Z_c axis coincides with the optical axis. The (X_c, Y_c, Z_c) axes form a right-hand triplet.
- $\Sigma_{u,v}$, 2D image reference system centered at O', the intersection point between optical axis and image plane π (the CCD surface). This plane is assumed to be parallel to the (X_c, Y_c) plane and at a distance f to the origin O_c, where f represents the effective focal length of the camera.
- $\Sigma_{r,c}$, 2D image reference system where the points coordinates are computed according to row and column number of corresponding pixel for the discrete image in the frame memory. The origin is located at the upper left corner of the image plane π.

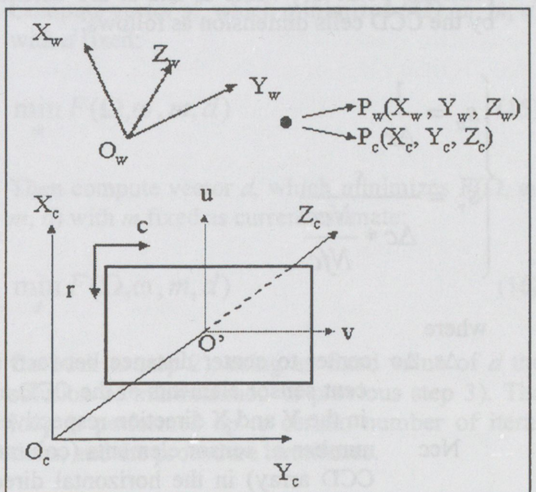

Fig. 1 : The adopted reference systems

Adopting a *pin-hole* camera model, the relationships between the 3D coordinates of target points and the corresponding 2D image coordinates can be defined as follows:

1) Rototraslation, transforming (X_w, Y_w, Z_w) coordinates of target point P in Σ_w, in the camera coordinates (X_c, Y_c, Z_c).

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R * \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \qquad (1)$$

where R is the rotation matrix defined by roll, pitch and yaw angles, while T is the traslation vector denoted by (T_x, T_y, T_z).

2) Perspective trasformation, that gives the undistorted position of P onto the image plane.

$$\begin{cases} u = f * \dfrac{X_c}{Z_c} \\ v = f * \dfrac{Y_c}{Z_c} \end{cases} \tag{2}$$

3) Change of image reference system in order to relate the metric image coordinates (u,v) of point P with the corresponding pixel coordinates (r,c) in the digitized image.

$$\begin{cases} r - r_0 = s_u * u \\ c - c_0 = s_v * v \end{cases} \Rightarrow \begin{cases} u = \dfrac{r - r_0}{s_u} \\ v = \dfrac{c - c_0}{s_v} \end{cases} \tag{3}$$

where (r_0, c_0) denotes the pixel position of the principal point O', while s_u and s_v are determined by the CCD cells dimension as follows:

$$\begin{cases} s_u = \dfrac{1}{\Delta r} \\ s_v = \dfrac{s}{\Delta c * \dfrac{Ncc}{Nfc}} \end{cases} \tag{4}$$

where

- Δr, Δc center to center distance between adjacent sensor elements of the CCD array, in the Y and X direction respectively;
- Ncc number of sensor elements (columns of CCD array) in the horizontal direction (Y axis);
- Nfc number of pixels in a line as sampled by the frame-grabber;
- s image scale factor, this is an additional uncertainty parameter introduced to take into account various source of error in the CCD array sampling, performed by the frame-grabber [3].

As regards the lens distortion, in our camera model we considered three major kind of lens distortions namely: radial, decentering and thin-prism. However, the calibration procedure was implemented in such a way to incorporate eventually further geometrical distortions, although this lead to a more complex camera model and requires an higher computational effort.
The corresponding set of distortion parameters that we have adopted, is reported below:

1) *Radial distortion*:

$$\begin{cases} \delta_{ur} = k_1 * u(u^2 + v^2) + k_2 * u(u^2 + v^2)^2 \\ \delta_{vr} = k_1 * v(u^2 + v^2) + k_2 * v(u^2 + v^2)^2 \end{cases} \tag{5}$$

2) *Decentering distortion*:

$$\begin{cases} \delta_{ud} = p_1 * (3u^2 + v^2) + 2p_2 * uv \\ \delta_{vd} = 2p_1 uv + p_2 * v(u^2 + 3v^2) \end{cases} \tag{6}$$

3) *Thin-prism distortion*:

$$\begin{cases} \delta_{up} = s_1(u^2 + v^2) \\ \delta_{vp} = s_2(u^2 + v^2) \end{cases} \tag{7}$$

4) *Total distortion*: when all the above distortions are present, the effective distortion can be modeled by addition of the corresponding expressions [3]. Therefore combining (5), (6), (7) we obtain the total amount of lens distortions along the *u* and *v* axes,

$$\begin{cases} \delta_{ut} = k_1 * u(u^2 + v^2) + k_2 * u(u^2 + v^2)^2 + p_1 \\ \qquad * (3u^2 + v^2) + 2p_2 * uv + s_1(u^2 + v^2) \\ \delta_{vt} = k_1 * v(u^2 + v^2) + k_2 * v(u^2 + v^2)^2 \\ \qquad + 2p_1 uv + p_2 * (u^2 + 3v^2) + s_2(u^2 + v^2) \end{cases} \tag{8}$$

Taking into account the distortion along the *u* and *v* axes, the relationship between distortion free image coordinates (u,v) and its corresponding pixel locations (r,c) becomes

$$\begin{cases} u + \delta_u(u,v) = \dfrac{r - r_0}{s_u} \\ v + \delta_v(u,v) = \dfrac{c - c_0}{s_v} \end{cases} \tag{9}$$

where $\delta_u(u,v)$, $\delta_v(u,v)$ can represent the total distortion or a combination of the above mentioned factors, according to the purposes of the calibration.
Note that the lens distortions are computed according to image coordinates (u,v) that are unknown.
Now, following the procedure presented by [5], if we introduce the new variables (u',v'), that represent the distorted location of image projections of target points onto a normalized image plane (Z_c=1),

$$\begin{cases} u' = \dfrac{u + \delta_u(u,v)}{f} = \dfrac{r - r_0}{f_u} \\ v' = \dfrac{v + \delta_v(u,v)}{f} = \dfrac{c - c_0}{f_v} \end{cases} \tag{10}$$

and replace in (10) the arguments of modeled distortions by u',v', we obtain the relationship between undistorted unknown image coordinates (u,v) and the distorted, but known, image coordinates (u',v') :

$$\begin{cases} \dfrac{u}{f} = u' + \delta_{u'}(u', v') \\ \dfrac{v}{f} = v' + \delta_{v'}(u', v') \end{cases} \tag{11}$$

The complete camera model is reported in (12) at the bottom of the page, where one can note that the full expression is linear respect to the considered distortion coefficients: this will simplify their estimation.

3. THE CALIBRATION ALGORITHM

Denoting with m the set of internal and external undistortion parameters

$$m = (r_0, c_0, f, s, T_x, T_y, T_z, \alpha, \beta, \gamma)$$

(where α,β and γ denote the three indipendent parameters of rotation matrix R) and with d the set of distortion parameters

$$d = (k_1, k_2, p_1, p_2, s_1, s_2)$$

in order to perform a camera calibration we have to determine the optimal estimate of parameters vectors m and d, given a set of visible target points ($X_{w,i}$, $Y_{w,i}$, $Z_{w,i}$) and the set of corresponding pixel locations (r'_i, c'_i). Due to the noise and the lens distortions that affect the image-points positions, the optimal estimate means computing the set of calibration parameters (m*,d*) which minimize the following merit function:

$$F(\Omega, \omega, m^*, d^*) = \min_{m,d} F(\Omega, \omega, m, d) \tag{13}$$

where Ω and ω represent the two sets of target and corresponding image points respectively.
In our case, we considered as objective function F the sum of squared discrepancy between the image coordinates (r_i, c_i), computed by features extraction algorithm, and the corresponding coordinates [$r_i(m,d)$, $c_i(m,d)$] as derived by 3D coordinates of target points ($X_{w,i}$, $Y_{w,i}$, $Z_{w,i}$) and the parameters (m,d), defining the camera model:

$$\sum_{i=1}^{N} \left\{ \left[r_i(m,d) - r_i \right]^2 + \left[c_i(m,d) - c_i \right]^2 \right\} \tag{14}$$

As the calibration parameters are related by nonlinear relationship (12), the minimization of (14) has to be performed through a nonlinear optimization algorithm, which will converge provided that a good initial guess of the parameters themselves is available. In order to meet these requirements, we approached the calibration process by combining the procedures proposed in [3] and [5], as previously mentioned in section 1.
The main structure is based on two-steps method like in [5], which can be summarized as follows:

1) First consider a distortion free camera model(d=0). To this purpose only image central points are used because they are less affected by lens distortions;
2) Compute vector m, which minimizes $F(\Omega, \omega, m, d)$ with d fixed:

$$\min_{m} F(\Omega, \omega, m, d) \tag{15}$$

3) Then compute vector d, which minimizes $F(\Omega, \omega, m, d)$ with m fixed as current estimate:

$$\min_{d} F(\Omega, \omega, m, d) \tag{16}$$

4) Go back to step 2), using as fixed value of d the solution of minimization in previous step 3). The loop is performed up to certain number of iterations, and the procedure terminates.

Referring to this method, we have introduced following modifications:

$$\begin{cases} r' = r_0 + f_u \left(\dfrac{r_{11} X_w + r_{12} Y_w + r_{13} Z_w + T_x}{r_{31} X_w + r_{32} Y_w + r_{33} Z_w + T_z} \right) - f_u \big[k_1 u' (u'^2 + v'^2) + k_2 u' (u'^2 + v'^2)^2 + 2 p_2 u' v' \\ \qquad + p_1 (3u'^2 + v'^2) + s_1 (u'^2 + v'^2) \big] \\ c' = c_0 + f_v \left(\dfrac{r_{21} X_w + r_{22} Y_w + r_{23} Z_w + T_y}{r_{31} X_w + r_{32} Y_w + r_{33} Z_w + T_z} \right) - f_v \big[k_1 v' (u'^2 + v'^2) + k_2 v' (u'^2 + v'^2)^2 + 2 p_1 u' v' \\ \qquad + p_2 (u'^2 + 3v'^2) + s_2 (u'^2 + v'^2) \big] \end{cases} \tag{12}$$

a) Non-linear optimization of vector m=(r_0,c_0,f,s,**R**,**T**) at step 2) is accomplished by iterative Levenberg-Marquardt algorithm [4]. As initial guess for m, at first iteration we use the value m', resulted from step 1), while in following iterations we use the value m'', obtained as output of step 2) in previous loop.

b) In the second part of the procedure, where *all* the image points are used to estimate also the distortion parameters, we perform again the non-linear estimate of m through L-M algorithm, while for vector d we carrie out a linear estimate based on solution of (12), with (k_1, k_2, p_1, p_2, s_1, s_2) as unknows.

c) After about four loops, all camera parameters are estimated together simultaneously by non-linear L-M algorithm, obtaining the optimal vector estimates m_{opt} and d_{opt}.

The overall scheme of the adopted procedure is showed in Fig. 2 and 3.

We have adopted the Levenberg-Marquardt iterative algorithm principally because it moves smoothly between two others widely used minimization methods, the *Steepest Descent* and the *Hessian*, combining them into one simple equation. In this way the algorithm can converge to true minimum, switching at each iteration between the two methods on the base of a changing threshold value λ [4]. We retain that this property can be succesfully used in a calibration procedure, since we deal with initial guesses about which we don't know how close to the true solution they are.

Such "switch" behavior can therefore resolve the problem of the "goodness" of initial estimates in minimizing the objective function F.

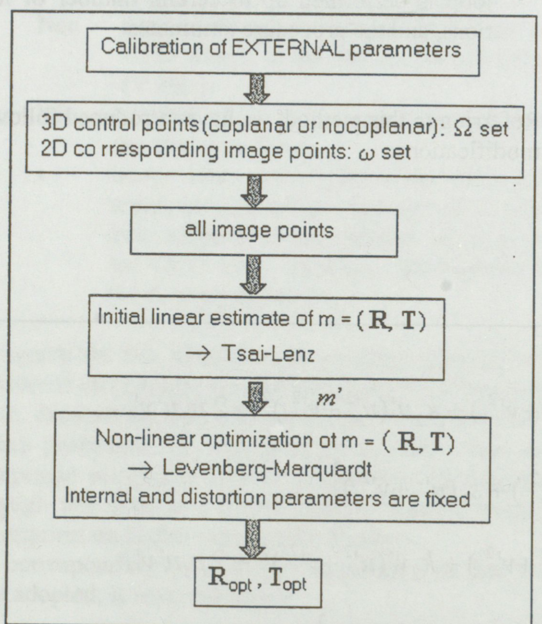

Fig. 2 : Flowchart of the estimation procedure of external parameters only.

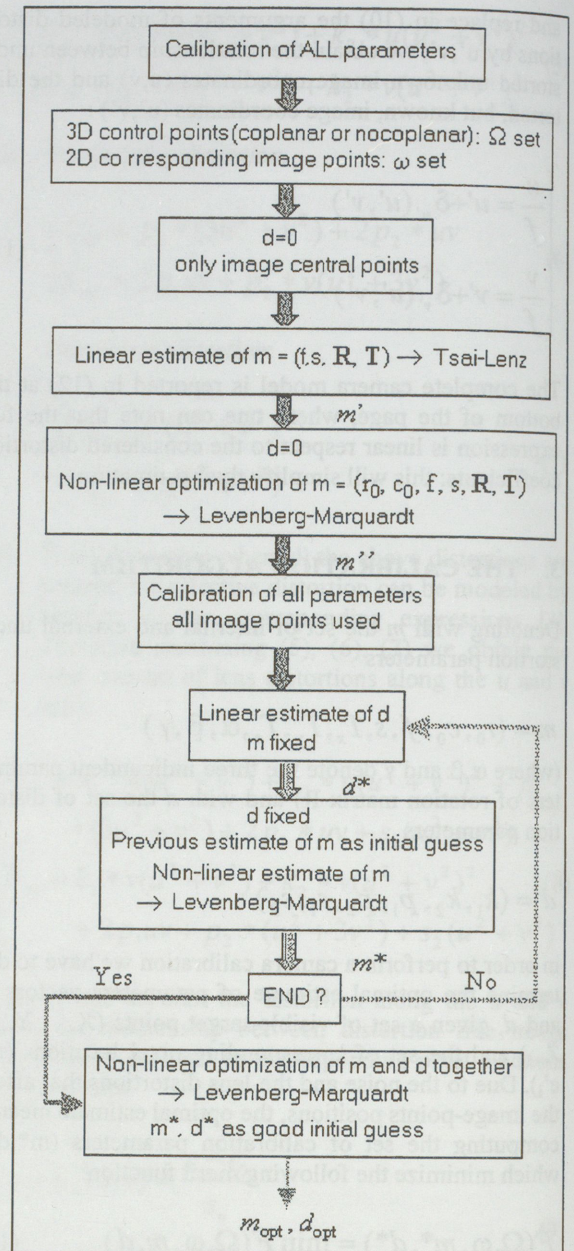

Fig. 3: Flowchart of calibration procedure of *all* parameters.

As reported in Fig. 2 and 3, our calibration procedure can be applied both to a set of coplanar (Z_w=0) and non-coplanar target points, so as it can be used to perform a full camera calibration or to evaluate only external parameters. In the last case the internal and distortion parameters of previous calibration are employed.

Since the initial linear estimate of vector m is based on Tsai-Lenz method, we defer the reader to references for more detailed explanations on it, while we provide a short overview on the application of Levenberg-Marquardt algorithm and on the linear estimate of distortion parameters.

Basically a maximum likelihood estimate of the model parameters is obtained minimizing the quantity

$$\chi^2(\mathbf{a}) = \sum_{i=1}^{N} \left[\frac{y_i - y(x_i;\mathbf{a})}{\sigma_i} \right]^2 \tag{17}$$

called the "chi-square", where y_i are the measured values, $y(x_i,\mathbf{a})$ are the values derived by model and vector parameters $\mathbf{a}=[a_1,a_2,\ldots,a_M]$, while σ_i denote the stdev of errors on y_i values (if unknown they can be all set to 1). In our case this chi square quantity is replaced by (14), in which (r_i,c_i) are the measured values while (r'_i,c'_i) are the image coordinates computed by camera model with lens distortions.

The gradient of χ^2 with respect to the parameters **a** has following components

$$\frac{\partial \chi^2}{\partial a_k} = -2\sum_{i=1}^{N} \frac{[y_i - y(x_i;\mathrm{a})]}{\sigma_i^{\,2}} \frac{\partial y(x_i;\mathrm{a})}{\partial a_k} \tag{18}$$

$$k = 1,2,\ldots,M$$

while additional partial derivative components are (neglecting second derivative):

$$\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} = 2\sum_{i=1}^{N} \frac{1}{\sigma_i^{\,2}} \left[\frac{\partial y(x_i;\mathrm{a})}{\partial a_k} \frac{\partial y(x_i;\mathrm{a})}{\partial a_l} \right] \tag{19}$$

$$k = 1,2,\ldots,M$$

Removing the factors of 2 from (18) and (19) by defining

$$\beta_k = -\frac{1}{2}\frac{\partial \chi^2}{\partial a_k} \qquad \alpha_{kl} = \frac{1}{2}\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} \tag{20}$$

the equations of Steepest Descent and Hessian minimization procedures, derived in terms of these factors, can be combined in a unique expression (22) if we define a new matrix α' of partial derivative of χ^2 [4]:

$$\alpha'_{jj} = \alpha_{jj}(1+\lambda)$$

$$\alpha'_{jk} = \alpha_{jk} \qquad (j \neq k) \tag{21}$$

$$\sum_{i=1}^{M} \alpha'_{ki}\,\partial a_i = \beta_k \tag{22}$$

where δa_i are the increments that added to the current approximation, give us the next one. When λ is very large, the matrix α' becomes diagonally dominant, so (22) goes over to be identical to Steepest Descent minimization formula, while as λ approaches zero, the equation (22) goes over the Hessian matrix formula. Given an initial estimate for the set of parameters **a**, the steps of Marquardt algorithm are as follows:

a) Compute $\chi^2(\mathbf{a})$;
b) Choose a modest value for λ, say λ=0.001;
c) Solve (22) for $\delta\mathbf{a}$ and evaluate $\chi^2(\mathbf{a} + \delta\mathbf{a})$;
d) If $\chi^2(\mathbf{a} + \delta\mathbf{a}) \geq \chi^2(\mathbf{a})$, increase λ, i.e. by a factor of 10, and go back to c);
e) If $\chi^2(\mathbf{a} + \delta\mathbf{a}) < \chi^2(\mathbf{a})$, decrease λ, i.e. by a factor of 10, update the current solution $\mathbf{a} \leftarrow \mathbf{a} + \delta\mathbf{a}$, and go back to c).

As regards the linear estimate of distortion parameters, this step is accomplished by least square solution of linear system **Ax** = **b** resulting from (8), (9) and (12). Given the coordinates of N control points, both in the world reference system Σ_w and in the image plane reference system $\Sigma_{u,v}$, **A** becomes the 2n x 6 matrix, containing the coefficients of distortion parameters, and **b** the 2n-dimensional vector of known terms, as reported below at the bottom of the page:

$$A = \begin{bmatrix} s_u \left[u'_1(u'^2_1 + v'^2_1) \;\; u'_1(u'^2_1 + v'^2_1)^2 \;\; (3u'^2_1 + v'^2_1) \;\; 2u'_1 v'_1 \;\; (u'^2_1 + v'^2_1) \;\; 0 \right] \\ s_v \left[v'_1(u'^2_1 + v'^2_1) \;\; v'_1(u'^2_1 + v'^2_1)^2 \;\; 2u'_1 v'_1 \;\; (3u'^2_1 + v'^2_1) \;\; 0 \;\; (u'^2_1 + v'^2_1) \right] \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{bmatrix} \tag{23}$$

$$x = \begin{bmatrix} k_1 \\ k_2 \\ p_1 \\ p_2 \\ s_1 \\ s_2 \end{bmatrix}; \quad b = \begin{bmatrix} r_0 + s_u * \left(\dfrac{r_{11}X_{wi} + r_{12}Y_{wi} + r_{13}Z_{wi} + T_x}{r_{31}X_{wi} + r_{32}Y_{wi} + r_{33}Z_{wi} + T_z} \right) - r_i \\ c_0 + s_v * \left(\dfrac{r_{21}X_{wi} + r_{22}Y_{wi} + r_{23}Z_{wi} + T_y}{r_{31}X_{wi} + r_{32}Y_{wi} + r_{33}Z_{wi} + T_z} \right) - c_i \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{bmatrix} \tag{24}$$

where $$\begin{cases} u_i^{'} = \left(\dfrac{r_i - r_0}{f_u}\right) \\ v_i^{'} = \left(\dfrac{c_i - c_0}{f_v}\right) \end{cases} \tag{25}$$

are the measured distorted image coordinates of the N control points, as derived by egde detection algorithm. Obviously in case we consider a camera model with a reduced set of distortion parameters, the corresponding columns in matrix **A** have to be eliminated.

4. THE CALIBRATION SOFTWARE GUI

In order to make more easy and understandable the calibration procedure from teaching and user final point of view, we have provided the algorithm with a Graphic User Interface. It was implemented in Matlab, because this is a development environment best suited for mathematical applications and it don't requires a specific knowledge about programming language like Fortran, C or C++. Of course Matlab has own syntax, anyway a complex software can be structured through scripts readable with a common text editor, making therefore easy to manage and modify the software itself.

An example of the GUI is depicted in Fig. 5 showing the DATA window, where all input parameters can be set up, namely: CCD camera construction features (menu *Camera*), number of used target planes (up to 6, menu *Target*) and the calibration mode (menu *Option*), listing the distortion parameters taken into account.

The user can view the results of calibration process both in numeric form, through the PARAMETERS window selecting the *param* submenu (Fig.6), and in graphical form selecting the submenu *result*. The aim of using such graphic windows should allow the user to assess in easier way the quality and accuracy of the calibration.

5. TEST AND RESULTS

In order to evaluate the overall accuracy of our method, we have performed a calibration test using a target plane with 48 black squares on white background, each having lateral dimension of 50mm and horizontal and vertical spacing of 150mm (Fig. 7). The vertices of these squares were employed as control points.

To this aim the Canny edge detection algorithm [1] was applied to the target squares, then the corresponding lines were recovered from resulted edge points by cubic splines interpolation. Finally the image coordinates of the vertices were determined as the points of edge lines intersections.

For the test we used a Kodak DCS-410 professional digital camera, employing a full-frame CCD image measuring 1524x1012 pixels, with a lateral dimension of CCD cells of 9μm. The target plane, located at a distance of ≅ 2m from the camera, was taken from different points of view (up to 6 positions) in order to get a larger spatial information about the perspective transformations experienced by control points. Employing an objective of 24mm the focal length was set up to infinity and the diagfram to 11.

Considering each time a different combination of distortion parameters, we have therefore performed several calibrations, which results are listed in tables 1, 2 and 3. The values of internal and distortion parameters are reported in table 1, according to 4 calibration test (rad2 means the estimate of both radial distortion coefficients). Instead, in table 2 (stdr,stdc) represent the errors along rows (r) and columns (c) in $\Sigma_{r,c}$, between image points positions, as derived by features extraction procedure, and the image coordinates of same points, as computed by the model. As both position are affected by geometrical distortions of the camera, the discrepancy can be regarded as an estimate of the noise superimposed on the image. The following four parameters (mean X_w, mean Y_w, std X_w, std Y_w) represent the mean and the stdev of the position errors along X_w and Y_w axes in reference system Σ_w. These values are calculated by differences between measured 3D coordinates of control points and backprojected positions on the target of corresponding image points, which locations were corrected through the estimated camera model. Finally, in the same way, the means and stdev of position errors between coordinates points in the camera reference system Σ_C were computed, which results are listed in table 3.

Table 1: calibration results about internal and distortion parameters

Internal parameters	rad + dec	rad + thin	rad2 + dec + thin
r_0	521.13	504.79	517.34
c_0	768.25	757.54	758.51
f	23.95	23.95	23.97
s	0.99923	0.99925	0.99991
Distortion parameters			
k_1	0.12650	0.12597	0.16914
k_2	0	0	-0.69756
p_1	0.002332	0	0.001899
p_2	-0.001526	0	-0.001651
s_1	0	0.003053	0.000648
s_2	0	-0.001948	-0.001753

Table 2: position errors of backprojected control points

	rad + dec	rad + thin	rad2 + dec + thin
std-r	0.209	0.209	0.206
std-c	0.247	0.248	0.244
mean X_w	0.185	0.185	0.242
mean Y_w	0.232	0.232	0.322
std X_w	0.130	0.120	0.216
std Y_w	0.161	0.162	0.257

Table 3: Relative position errors of backprojected control points in reference system Σ_C.

	rad + dec	rad + thin	rad2 + dec + thin
mean X_c	1/15669	1/15647	1/11769
mean Y_c	1/13202	1/13172	1/9403
std X_c	1/22032	1/21990	1/12542
std Y_c	1/18618	1/18555	1/11295

As pointed out in tables 2 and 3, beside of performing a calibration with less or more distortion parameters, one should choose the camera model according to lower backprojection errors, which are represented by (mean X_w, meanY_w) and (std X_w, std Y_w) or, equivalently, by corresponding relative values computed in Σ_C, while (stdr, stdc) can give a feeling of superimposed noise.

CONCLUSIONS

Today, the camera calibration is becoming even more an important issue in the digital photogrammetry field. In the same time non-metric photocameras are widely spreading on the market, due their relative low price and improvement of image quality, that is sufficient for image processing. In order to put in touch these two fields, in the ambit of the Photogrammetry course we have developed an alternative calibration method, in which the Tsai-Lenz and Cohen-Herniou algorithms were properly revised. The procedure was then implemented as teaching software, in which a graphic interface helps the user for data input, evaluation of calibration parameters results and assessement of its accuracy. The proposed algorithm is able to taken into account various combinations of lens distortions, allowing the user to choose the best distortion model according to the application field and calibration requirements.

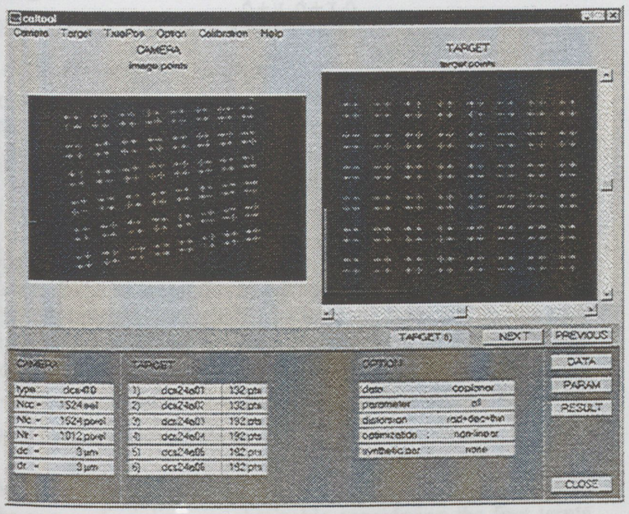

Fig. 5 : DATA window for calibration parameters input

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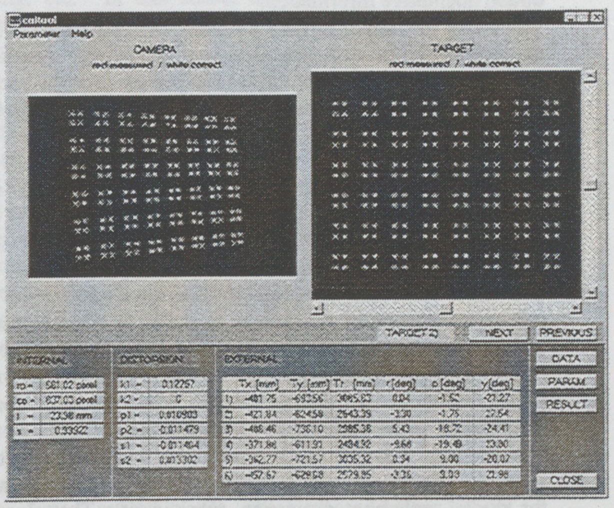

Fig. 6 : The PARAMETERS window showing the calibration results.